

**TUTORIAL SOLUTION: September 29, 2021**

1. a.  $P(\neg E, \neg S, \neg M, \neg B) = P(\neg E) P(\neg M) P(\neg S \mid \neg E, \neg M) P(\neg B \mid \neg M)$   
 $= (1 - 0.4) * (1 - 0.1) * (1 - 0.1) * (1 - 0.1) = 0.4374$

b.  $P(B) = P(B \mid M) P(M) + P(B \mid \neg M) P(\neg M)$   
 $= 1.0 * 0.1 + 0.1 * (1 - 0.1) = 0.19$

c.  $P(M \mid B) = P(B \mid M) P(M) / P(B)$   
 $= (1.0)(0.1) / (0.19) \cong 0.5263$

d.  $P(M \mid S, B, E) = P(M, S, B, E) / P(S, B, E)$   
 $= P(M, S, B, E) / [ P(M, S, B, E) + P(\neg M, S, B, E) ]$

$$P(M, S, B, E) = P(E) P(M) P(S \mid E, M) P(B \mid M) = 0.4 * 0.1 * 1.0 * 1.0 = 0.04$$

$$P(\neg M, S, B, E) = P(E) P(\neg M) P(S \mid E, \neg M) P(B \mid \neg M)$$

$$= 0.4 * 0.9 * 0.8 * 0.1 = 0.0288$$

$$\text{Therefore, } P(M \mid S, B, E) = 0.04 / [ 0.04 + 0.0288 ] = 0.5814$$

e.  $P(E \mid M) = P(E) = 0.4$

In the absence of evidence on S, the node S d-separates E and M. Therefore, E and M are conditionally independent.

2. a.

<b>F</b>	<b>C</b>	
T	T	$0.1 * 0.8 = 0.08$
T	F	$0.1 * 0.2 = 0.02$
F	T	$0.9 * 0.3 = 0.27$
F	F	$0.9 * 0.7 = 0.63$

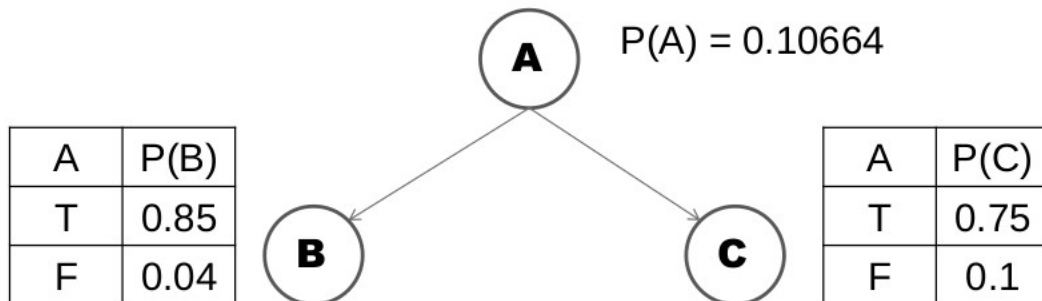
b. No; because  $P(C) = 0.35$  but  $P(C \mid F) = 0.8$

c.  $P(C) = 0.08 + 0.27 = 0.35$   
 $P(F \mid C) = P(F, C) / P(C) = 0.08 / 0.35 \cong 0.23$   
 $P(F \mid \neg C) = P(F, \neg C) / P(\neg C) = 0.02 / 0.65 \cong 0.03$

$$\begin{aligned}
 3. \quad P(A \mid \neg B, C) &= P(\neg B, C \mid A) P(A) / P(\neg B, C) \\
 &= P(\neg B, C \mid A) P(A) / (P(\neg B, C \mid A) P(A) + P(\neg B, C \mid \neg A) P(\neg A))
 \end{aligned}$$

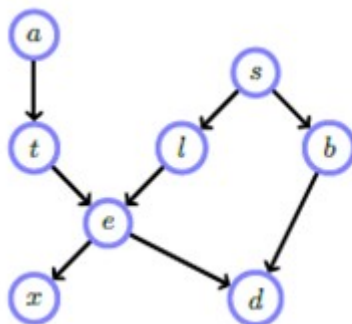
$$\text{Now, } P(\neg B, C \mid A) = P(\neg B \mid A) P(C \mid A) = (0.15)(0.75) = 0.1125$$

$$P(\neg B, C \mid \neg A) = P(\neg B \mid \neg A) P(C \mid \neg A) = (0.96)(0.1) = 0.096$$



$$\begin{aligned}
 \text{Therefore, } P(A \mid \neg B, C) &= (0.1125 * 0.10664) / ((0.1125 * 0.10664) + (0.096 * 0.89336)) \\
 &= 0.1227
 \end{aligned}$$

4.



*x* = Positive X-ray  
*d* = Dyspnea (Shortness of breath)  
*e* = Either Tuberculosis or Lung Cancer  
*t* = Tuberculosis  
*l* = Lung Cancer  
*b* = Bronchitis  
*a* = Visited Asia  
*s* = Smoker

The table values are:

$$\begin{array}{ll}
 P(a) = 0.01 & P(s) = 0.5 \\
 P(t|a) = 0.05 & P(t|\neg a) = 0.01 \\
 P(l|s) = 0.1 & P(l|\neg s) = 0.01 \\
 P(b|s) = 0.6 & P(b|\neg s) = 0.3 \\
 P(x|e) = 0.98 & P(x|\neg e) = 0.05
 \end{array}$$

$$P(d|e,b) = 0.1$$

$$P(d|e,\neg b) = 0.7$$

$$P(d|\neg e,b) = 0.8$$

$$P(d|\neg e,\neg b) = 0.1$$

$P(e|t,l) = 0$  only if both  $t$  and  $l$  are false, and 1 otherwise.

$$P(\neg s) = 0.5$$

$$P(\neg a) = 0.99$$

$$P(d) = P(d,e,b) + P(d,\neg e,b) + P(d,e,\neg b) + P(d,\neg e,\neg b)$$

$$P(d,e,b) = P(d|e,b) P(e,b) = P(d|e,b) P(e) P(b)$$

$$P(e) = P(e,t,l) + P(e,\neg t,l) + P(e,t,\neg l) + P(e,\neg t,\neg l)$$

$$P(e,t,l) = P(e|t,l) P(t,l) = P(e|t,l) P(t) P(l)$$

$$P(t) = P(t|a) P(a) + P(t|\neg a) P(\neg a) = 0.05 * 0.01 + 0.01 * 0.99 = 0.0104$$

$$P(\neg t) = 1 - P(t) = 1 - 0.0104 = 0.9896$$

$$P(l) = P(l|s) P(s) + P(l|\neg s) P(\neg s) = 0.1 * 0.5 + 0.01 * 0.5 = 0.055$$

$$P(\neg l) = 1 - P(l) = 1 - 0.055 = 0.945$$

$$P(e) = P(e|t,l) P(t) P(l) + P(e|\neg t,l) P(\neg t) P(l) + \\ P(e|t,\neg l) P(t) P(\neg l) + P(e|\neg t,\neg l) P(\neg t) P(\neg l)$$

$$= 1 * P(t) P(l) + 1 * P(\neg t) P(l) + 1 * P(t) P(\neg l) + 0 * P(\neg t) P(\neg l)$$

$$= P(t) P(l) + P(\neg t) P(l) + P(t) P(\neg l) = 1 - P(\neg t) P(\neg l)$$

$$\{\text{Since, } P(t) P(l) + P(\neg t) P(l) + P(t) P(\neg l) + P(\neg t) P(\neg l) = 1\}$$

$$= 1 - 0.9896 * 0.945 = 0.064828$$

$$P(\neg e) = 1 - 0.064828 = 0.935172$$

$$P(b) = P(b|s) P(s) + P(b|\neg s) P(\neg s) = 0.6 * 0.5 + 0.3 * 0.5 = 0.45$$

$$P(\neg b) = 1 - P(b) = 1 - 0.45 = 0.55$$

$$P(d) = P(d|e,b) P(e) P(b) + P(d|\neg e,b) P(\neg e) P(b) + \\ P(d|e,\neg b) P(e) P(\neg b) + P(d|\neg e,\neg b) P(\neg e) P(\neg b)$$

$$= 0.1 * 0.064828 * 0.45 + 0.8 * 0.935172 * 0.45 + \\ 0.7 * 0.064828 * 0.55 + 0.1 * 0.935172 * 0.55$$

$$= 0.41597242$$